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# Yardstick Competition in Presence of Shocks: A Spatial Voting Model Approach

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## Abstract

I analyse a yardstick competition game using a spatial voting model, where voters vote for a candidate according to the distance between their Ideal Point and the policy selected by a candidate. The policy which is closest to a voter's IP provides the voter with a higher utility so that minimizing the distance means maximising the utility. I demonstrate that in the presence of asymmetrical information the existence of yardstick competition entails a selection device but not a discipline device, suggesting the existence of a trade off between these two goals. In the second part, I analyse an economic environment characterised by the presence of shocks, whose sign and magnitude are a private information of incumbents. This time, the introduction of yardstick competition acts both as a selection and a discipline device.

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# 1 Introduction

Is yardstick competition beneficial for voters' welfare? This question still has not found a clear answer in the literature and depends on another interesting question. Are governments benevolent or Leviathan? Economists who believe that governments are welfaristic, naturally tended to assume that results in the economy are efficient and, as a consequence, inter-governmental yardstick competition generates negative externalities that lower the total welfare. Otherwise, some other economists who believe that governments are rent-seekers suggest that yardstick competition may act as a discipline device and thus is beneficial. In a recent work Oll ([16]) argued in his introduction: "In a decentralised tax system, a means of demonstrating to voters that a tax increase is necessary is to show that taxes are higher elsewhere for the same benefits provided" and "by threatening to punish at the pools local officials imposing tax rates out of line with other jurisdictions, voters compel incumbents to look at other localities' taxing behavior when determining taxes, which sets the stage for tax-mimicking behavior". Actually, those seem rather two extreme positions. It is more realistic to imagine a world populated by both good and bad governments. This assumption allows for the possibility to consider a game theoretic environment where the two types of government take their decisions, taking into account the existence of other types of government. A complication derives from the existence of adverse selection and moral hazard issues which prevent to identify the true nature of a government and whether policies undertaken are the right ones. Thus, the key point is to evaluate whether the yardstick competition is able to reduce the negative effects of the asymmetrical information; in other words it would be crucial to understand whether a yardstick competition is able both to achieve a *selection effect*, achievable when a type of government is forced to reveal itself as his true type, and a *discipline effect*, achievable when the bad government refrain itself to divert rents away from voters' welfare. Many works have provided theoretical support and empirical evidence, but up to now there is no clear evidence about effects the yardstick competition entails. A theoretical and empirical example was originally developed by Besley and Case ([6]), who show that in the presence of agency problems where voters are able to appraise the incumbents' relative performance, different equilibria may arise. For instance, when there exist both the good and the bad type of government, the yardstick competition entails a selection effects (the bad type prefers immediately to divert resources in the first period knowing he will lose the elections). In another paper, Basley & Smart ([3]) argue that the yardstick competition is likely to be welfare improving for voters when it is more likely that politicians are benevolent and bad for welfare when it is most likely that politicians are Leviathan. Thus, the results are still ambiguous, depending on the parameter of the model. Nevertheless, it seems that the yardstick competition

enforces more the selection than the discipline effect. On the negative side are also Bordignon et al. ([7]), which used a theoretical tax-setting model which demonstrates how the yardstick competition may induce more pooling or separating behavior among different types of government, depending on model parameters.

The goal of this paper is twofold: first of all, it analyses a typical yardstick competition game from a new perspective represented by a *spatial voting model*. Spatial voting models have been deeply studied in the literature from a theoretical perspective<sup>2</sup>, but strange enough these studies still have not found many applications in real economic problems. One of the main assumptions of a spatial voting model is that individuals have an Ideal Point (IP) in a multi issue space, which represents the point where they get the maximum satisfaction. As a consequence, it is natural to think they vote for a candidate who sets the closest policy to their IP. Roughly speaking, they vote according to the distance between the ideal policy they have in mind and the real policy a government sets. This conjecture is subtle. Usually, in Bayesian games, we have good and bad types of individuals. The concept of “good” or “bad” is a-priori determined. A good type is who pursues the society’s interest and the bad is who pursues its own interests. In a spatial voting model the concept of “goodness” or “badness” is more relative. Since the exclusive criterion agents use to evaluate a politician is represented by the *distance*, they perceive a candidate as good if that candidate chooses a closer policy to their IP than another candidate, who is automatically perceived as bad. Thus, voters tend to evaluate candidates according to a very egoistic criterium: they only vote candidates who they feel very close to their needs. This is not far from what happens in the real world. Everyone of us casts his vote evaluating the “distance” he feels between his position and candidate’s position. Notice that the distance perceived may be due to several reasons: one may decide to vote for a candidate because he evaluates that he will provide him more money transfers or may vote for it because he feels that the *ideological* distance is small. A voter may adopt different criteria of evaluation, but the important thing here is that he is always able to translate his evaluation in a metric distance, and vote for a candidate whose distance from his IP is the minimum. Mathematically speaking, we are solving a duality problem: minimising the distance between the voter’s IP and the candidate’s policy means maximising the voter’s utility. Secondly, referring to the resolution of the yardstick game, it will be demonstrated that in a situation where voters are not able to evaluate the type of incumbent and the type of shock which affects the economy but they perfectly evaluate the outcome of policies of different jurisdictions, equilibria may be both pooling and separating. Thus, the yardstick competition may exacerbate the will-

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<sup>2</sup>the literature on spatial voting models is huge: see Hinich & Ordeshook, 1970 ([13]), Davis, DeGroot & Hinich, 1972 ([10]), Hinich, 1977 ([12]), Hinich & Pollard, 1981 ([14])

ingness of bad candidates of immediately separate from good candidates. As a consequence the yardstick competition contributes to select good and bad politicians but also acts an incentive device to promote efficiency. The paper is organized as follows: section 1 briefly introduce the voting model literature, section 2 analyses the general features of the model, section 3 analyses a model characterized by complete information, section 4 and 5 analyses the same model in which there exists asymmetrical information both in the absence and in the presence of yardstick competition, section 6 and 7 adds the presence of shocks in the model, section 8 concludes. Finally, Appendix A provides a technical discussion of the most important lemmas of spatial voting models and Appendix B provides an analysis of the multivariate case.

## 2 The model

### 2.1 Candidates

I consider a two-period model with three political candidates D,R and W, where D stands for Democratic, R for Republican and W for Welfaristic. On each party I attached a political label, which I assume is exogenously taken at the beginning of the electoral campaign. For instance, we may think about the most familiar labeling system in the U.S., where candidates are located in a left-right or liberal-conservative scale. In our model we assume that candidate R is labeled as “conservative” and that, in a very simplified vision, it is supported by voters who get higher utility in the taxation of labor rather than capital. Otherwise, the candidate D is labeled as “liberal” and he supports voters who get higher utility in the taxation of capital rather than labor. Thus, the space of candidates is given by  $\Theta^C = \{D, R, W\}$ . Furthermore candidate D may be a *good* type or a *bad* type and so may be candidate R. Thus, the space of type is  $\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}\}$ . Each candidate may play only 4 policies: two populist policies (the right-wing populist policy  $a^R$  and the left-wing populist policy  $a^D$ ), which provide more welfare to oriented voters, a welfaristic policy  $a^W$ , which is neutral and a bad policy  $a^{bp}$  which enables the government to subtract rents to the society’s welfare.

#### 2.1.1 Candidate’s preferences

Each candidate has the following preferences ordering:

Candidate D good type:  $a^D \succ a^W \succ a^R \succ a^{bp}$

Candidate D bad type:  $a^{bp} \succ a^D \succ a^W \succ a^R$

Candidate R good type:  $a^R \succ a^W \succ a^D \succ a^{bp}$

Candidate R bad type:  $a^{bp} \succ a^R \succ a^W \succ a^D$

These preference orderings may be represented by an utility function  $r$ , such

that  $a^x \succeq a^y \Leftrightarrow r(a^x) \geq r(a^y)$ . Thus, in utility terms, candidate's preferences may be represented in the following manner:

$$\text{Candidate D good type: } r(a^D) > r(a^W) > \overbrace{r(a^R)}^{=0} > r(a^{bp})$$

$$\text{Candidate D bad type: } r(a^{bp}) > r(a^D) > \overbrace{r(a^W)}^{=0} > r(a^R)$$

$$\text{Candidate R good type: } r(a^R) > r(a^W) > \overbrace{r(a^D)}^{=0} > r(a^{bp})$$

$$\text{Candidate R bad type: } r(a^{bp}) > r(a^R) > \overbrace{r(a^W)}^{=0} > r(a^D)$$

To facilitate calculations, I assume the existence of a strict preferences ordering (the inequality is strict) and that the third term of the ordering is normalized to zero.

## 2.2 Voters

I suppose the existence of a population of voters, portioned in three equal groups (i.e. 1 voter per group): the welfarist voters, who are those who have not any particular preference for a candidate, candidate D-oriented voters, who support the Democratic Party and candidate R-oriented voters, who support the Republican Party.

### 2.2.1 Voter's preferences

Each voter has the following preferences ordering:

D-oriented Voter:  $a^D \succ a^W \succ a^R \succ a^{bp}$

R-oriented Voter:  $a^R \succ a^W \succ a^D \succ a^{bp}$

Welfarist Voter:  $a^W \succ a^D \vee a^R \succ a^{bp}$

Voters observe candidate policies and cast their vote for the candidate who has selected the nearest policy to their IP. In Appendix A I provide a technical explanation of the mechanism adopted by voters to select which is the candidate who provide them the highest utility.

## 3 A complete information case

Suppose a two-period game where the population is represented by only three voters (D,R,W) and two candidates (D,R) which duel to obtain the power. Suppose also that the economy is not affected by shocks. Without loss of generality, I assume that party R is the incumbent and party D is the challenger. Each candidate has to choose a vector of policy in the two-dimension Cartesian space  $\mathbb{R}^2$ ; the vector of policies encompasses the marginal tax rate on labor and the marginal tax rate on capital and is represented by a singleton in the space. At the end of the mandate, elections

take place. Voters perfectly recognize the type of candidate, so that parties cannot exploit any information advantage, and they cast their vote to the party which chooses a policy vector closer to their IPs. [FIGURE 1 HERE] Figure 1 shows the IP of each voter (black singletons), while concentric circles represents voter's indifference curves. The smaller the circle, the nearer the policy chosen by a candidate to the voter's IP and the higher the utility the voter gets. Voters who support the Republican party have an ideal point such as R, where the tax rate on capital is equal to zero and the tax rate on labor is equal to one. Symmetrically, voters who support the Democratic party have an ideal point such as D, where the tax rate on labor is equal to zero and the tax rate on capital is equal to one. In the middle of the space, on W, are located welfaristic voters, that may be intended as "super-rational" voters who do not have any particular preference toward one of the two candidates.

Circles correctly represent the voters' utility preferences only if voters assign exactly the same weight to every dimension of the policy space. If not, the utility preference is given by a non circular structure, that indicates that the decrease in utility is not the same in every dimension. In my example we have a policy vector given by  $a = \{\tau_l, \tau_k\}$  in the Cartesian space  $\tau_l \times \tau_k$ ; we can represent voters' preferences with circles if and only if:  $\frac{\partial U(\tau_l, \tau_k)}{\partial \tau_l} = \frac{\partial U(\tau_l, \tau_k)}{\partial \tau_k}$ ; otherwise, if  $\frac{\partial U(\tau_l, \tau_k)}{\partial \tau_l} \neq \frac{\partial U(\tau_l, \tau_k)}{\partial \tau_k}$  voters' preferences are represented by ellipses, where the longer axis refers to that dimension which shows the higher decrease in the utility with respect to the other dimension, once we move away from the IP. Figure 2 sketches a typical situation where the decrease in utility is not the same in every direction. Furthermore, the rotation of the major and the minor axis of the elliptical contours of each citizen's loss function may be represented by a matrix of coefficients. If this matrix is equal for every voter, then we are in the "common orientation" case, and the ellipses have the same rotation axis.

[FIGURE 2 HERE]

Finally, voters' preferences are uniformly distributed (in a discrete case), while they are represented by a density function, say  $f(x)$ , which is *radial symmetric*<sup>3</sup> and *unimodal* (in a continuous case). Figure 3 shows the candidates' IPs. Initially, we assume that candidate IPs must only stand over the budget constraint line; this assumption may be realistic as candidates are Welfaristic with a slight preference toward one of the two ideological voters. Thus, candidates can only choose a policy which stands over the budget constraint line.

[FIGURE 3 HERE]

Furthermore, I assume that the party which gets the majority of votes win the elections; in the case of a tie a coin is tossed as to decide which party will take power. Finally the utility that voters get in the second period of

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<sup>3</sup>that is  $f(x) = f(-x)$

the game is discounted by a discount factor  $\beta \in [0, 1]$ .

**Proposition 1** *In an economy with a finite number of identical voters the two parties choose the vector of policies which exactly coincides with the Welfarist voter's IP, if and only if they discount the future sufficiently little; otherwise, they prefer to locate exactly on their preferred IPs.*

*Proof:* In the second period of the game, once the elections have taken place, the elected candidate chooses the policy which stands on its IP ( $a^D$  for party D and  $a^R$  for party R). Otherwise, in the first period, parties choose the policy which maximizes the expected utility over the two periods. Since the two parties can only play either a populist policy or a welfaristic policy, we can depict this situation of strategic interaction between the two parties by the meaning of the following game:

	$a^R$	$a^W$
$a^D$	$r(a^D) + \frac{1}{2}\beta r(a^D); r(a^R) + \frac{1}{2}r(a^R)$	$r(a^D); r(a^W) + \beta r(a^R)$
$a^W$	$r(a^W) + \beta r(a^D); r(a^R)$	$r(a^W) + \frac{1}{2}\beta r(a^D); r(a^W) + \frac{1}{2}\beta r(a^R)$

Rows show policies played by party D, while columns policies played by party R and the four boxes show payoffs of the game with the two parties' expected utilities. If party R plays  $a^R$  Party D plays  $a^D$  if and only if  $r(a^D|a^R) > r(a^W|a^R)$ . This happens when  $r(a^D) + \frac{1}{2}\beta r(a^D) > r(a^W) + \beta r(a^D)$  or when  $\beta \in [0, 2(1 - \frac{a^W}{a^D})]$ . If party R plays  $a^W$  Party D plays  $a^D$  if and only if  $r(a^D|a^W) > r(a^W|a^W)$ . This happens when  $r(a^D) > r(a^W) + \frac{1}{2}\beta r(a^D)$  or  $\beta \in [0, 2(1 - \frac{a^W}{a^D})]$ . The same holds for party R.

Then, we may distinguish three cases:

1. when  $\beta \in [0, 2(1 - \frac{a^W}{a^D})]$  the only one Nash equilibrium is one such that the two parties play a populist strategy which enables them to get the maximum utility; elections' outcome is a tie and a coin is tossed to decide which candidate will take power in the second period.
2. when  $\beta \in (2(1 - \frac{a^W}{a^D}), 1]$  the only one Nash equilibrium is one such that the two parties play the welfaristic policy which coincides with the welfaristic voter's IP; elections' outcome is again a tie.
3. when  $\beta = 2(1 - \frac{a^W}{a^D})$  the two parties are indifferent to play either  $a^D$  or  $a^W$  and then infinite equilibria in mixed strategies arise.

## 4 A game with incomplete information and absence of shocks

Suppose now to modify the space of types and consider the existence of two types of candidates both for the incumbent and for the challenger (say a *good* and a *bad* type). Thus, the space of types can be written



as  $\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}\}$ , where  $g$  denotes the good type and  $b$  the bad type. Furthermore, I introduce an asymmetry in information between the two parties, such that the incumbent does not know whether it is challenging a good or a bad opponent; it only knows that there exists an *a priori* probability equal to  $q$  that the challenger is a good type and a probability equal to  $1 - q$  that the challenger is a bad type. The distribution of probability is common knowledge between candidates and voters, in the sense that also voters know the existence of this a-priori probability that candidate is good (this probability may be seen as the reputation of the candidate). Figure 4 shows the policy Cartesian space  $\tau_l \times \tau_k$ , which can be normalized to  $1 \times 1$ . The straight line represents the Government budget constraint being equal to zero. With respect to the previous case, two additional IP point (GDb and GRb) have been added and they represent the IP of bad Governments (party D and R respectively). These two points represent a location where Governments can get a rent which is equal to the distance from any policy which stands in the North-East triangle above the budget constraint line and the budget constraint line. The longer this distance, the higher the rent subtracted by the Government.

[FIGURE 4 HERE]

I study two cases where the incumbent may be either a good or a bad type. In the first I study a case where the incumbent may face a good challenger which is “super-welfarist” in a sense that he always plays strategy  $a^W$  or a bad Government who can only play strategy  $a^{bp}$ . Thus, the game can be formalized in the following structure:

$$\Theta^C = \{D, R, W\}$$

$$\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}\}$$

$$A^I = A^D = \{a^D, a^R, a^W, a^{bp}\} \subseteq E_2$$

$$A^{Gg} = A^{Rg} = \{a^W\} \subseteq E_2$$

$$A^{Gb} = A^{Rb} = \{a^{bp}\} \subseteq E_2$$

$$\Pr(\theta^R = \theta^{Rg}) = q$$

$$\Pr(\theta^R = \theta^{Rb}) = 1 - q$$

In the second example the incumbent may face either a good challenger who can only play a conservative policy  $a^R$ , or a bad challenger who again can only play strategy  $a^{bp}$ . In this second case the problem is formalized as follows:

$$\Theta^C = \{D, R, W\}$$

$$\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}\}$$

$$A^I = A^D = \{a^D, a^R, a^W, a^{bp}\} \subseteq E_2$$

$$A^{Gg} = A^{Rg} = \{a^R\} \subseteq E_2$$

$$A^{Gb} = A^{Rb} = \{a^{bp}\} \subseteq E_2$$

$$\Pr(\theta^R = \theta^{Rg}) = q$$

$$\Pr(\theta^R = \theta^{Rb}) = 1 - q$$

Furthermore, there exists an asymmetric information between candidates and voters, since also voters are not able to evaluate whether a candidate is a good or a bad type. Nevertheless, the concept of goodness that candidates have differs to the concept of goodness that voters have. Candidates believe a candidate is good if he acts in the society's interest whilst he is bad if he acts exclusively in his own interest. Otherwise, voters believe a candidate is good if he undertakes a policy which is the closest to their IP, regardless of the overall effects which this policy has on society and believe a candidate is bad if he acts in his own interest. This concept of goodness differs with respect to the previous literature where it was seen in an absolute value (a politician is good if he does whatever to increase the welfare of society) whilst in this paper it is seen as a relative value (a politician is good if he does whatever to increase my welfare regardless to the others' welfare). This asymmetry in information is a typical agency problem. Voters are only able to appraise incumbents' performance, in the absence of yardstick competition and incumbents' *relative* performance in the yardstick competition case, (i.e. from the media). After having observed incumbents' performance, voters update their beliefs about the goodness of politicians and cast their vote in the elections according to these beliefs. Thus, the timing of the game is the following: first, Nature chooses incumbent type. Then, each incumbent announces (and commits) to a policy. Voters observe incumbents' announcements and update their beliefs using Bayes' rule. Finally, according to their updated beliefs they decide whether to reelect the incumbent or not. Denotating with  $\gamma$  ex-post beliefs, I assume that voters decide to reelect the incumbent if and only if  $\gamma \geq q$ . In this framework, it is very easy to see that voters attribute  $\gamma = 1$  if the policy of one candidate is nearer to their IP than the policy of the other candidate,  $\gamma = 0$  if the policy of one candidate is farer to their IP than the policy of the other candidate, and  $\gamma = \frac{1}{2}$  if the policy of one candidate is as near to their IP as the policy of the other candidate. I find perfect Bayesian equilibria of this tax-setting game.

**Proposition 2** *If the incumbent is a good type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ , whilst he plays the welfaristic strategy  $a^W$  if  $q \in (\frac{2}{\beta r(a^D)}(r(a^D) - r(a^W)), 1]$ .*

*Proof:*  $EU(a^D) = r(a^D) + \beta(1 - q)r(a^D)$  is always greater than  $EU(a^R) = r(a^R) + \beta(1 - q)r(a^R)$  and than  $EU(a^{bp}) = r(a^{bp}) + \frac{1}{2}\beta(1 - q)r(a^D)$ , whilst it is greater than  $EU(a^W) = r(a^W) + \frac{1}{2}\beta q r(a^D) + \beta(1 - q)r(a^D)$  for  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ <sup>4</sup>.

This result can be interpreted as follows: if the probability to face a good challenger is sufficiently small, the incumbent has a great opportunity to play a policy which stays on his IP. Otherwise, if the probability to face a good challenger is high, the incumbent realizes that playing his preferred policy would not be sufficiently safe to assure the re-election and then he prefers to play a welfaristic policy to get the welfaristic citizens' votes.

**Proposition 3** *If the incumbent is a bad type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^{bp})}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp}))]$ , whilst he plays the bad policy  $a^{bp}$  if  $q \in (\frac{2}{\beta r(a^{bp})}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})), 1]$*

*Proof:*  $EU(a^D) = r(a^D) + \beta(1 - q)r(a^{bp})$  is always greater than  $EU(a^R) = r(a^R) + \beta(1 - q)r(a^{bp})$  and  $EU(a^{bp}) = r(a^{bp}) + \frac{1}{2}\beta(1 - q)r(a^{bp})$  is always greater than  $EU(a^W) = r(a^W) + \frac{1}{2}\beta q r(a^{bp}) + \beta(1 - q)r(a^{bp})$ .  $EU(a^D)$  is greater than  $EU(a^{bp})$  for  $q \in [0, \frac{2}{\beta r(a^{bp})}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp}))]$ .

The result says that if the probability that the incumbent faces a good challenger is sufficiently low he plays the populist strategy, since the electorate is always able to see the bad strategy undertaken by the party (bad type) and reelect the incumbent, since it is not able to evaluate whether the type is good or bad. Otherwise, if the probability to face a good challenger is high, the bad incumbent realizes that it becomes too costly mimicking a good type (which would entail a pooling equilibrium) and so he plays the bad policy even though he knows that he will lose the elections. As a consequence a separating equilibrium arises. Analyse now the situation where the government (good type) is committed to play the populist strategy.

**Proposition 4** *If the incumbent is a good type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ , whilst he plays the welfaristic strategy  $a^W$  if  $q \in (\frac{2}{\beta r(a^D)}(r(a^D) - r(a^W)), 1]$*

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<sup>4</sup>A note: In all the proofs, the expression *always greater* which refers to the expected utility of a strategy with respect to another may be read as *that strategy whose expected utility is always greater than another strategy's expected utility strictly dominates that other strategy*.

*Proof:* same as before.

**Proposition 5** *If the incumbent is a bad type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^{bp})}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp}))]$ , whilst he plays the bad policy  $a^{bp}$  if  $q \in (\frac{2}{\beta r(a^{bp})}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})), 1]$*

*Proof:* same as before.

## 5 A case of yardstick competition

I introduce now the yardstick competition framework. The main goal is to verify how the introduction of another jurisdiction may act as a discipline device for candidates. I suppose that domestic voters are able to update their beliefs about the domestic government by comparing the policy it undertakes with that undertaken by the foreigner jurisdiction. I demonstrate that, due to this comparison, the bad domestic candidate refrains himself to mimic the good type, since the mimicking strategy would become too costly for him. As a consequence, we expect a reinforcement of separating equilibria and a weakening of pooling equilibria. I consider at first a foreign government which can be either a good or a bad type. If he is good, he plays a welfaristic strategy  $a^{WF}$ , whilst if he is bad, he plays a bad policy strategy  $a^{bpF}$ . Later on, I will consider a good government committed to play a populist strategy  $a^{RF}$ , instead of playing  $a^{WF}$ , whilst the bad government plays the bad policy strategy  $a^{bpF}$ . Thus, the game can be formalized in the following terms:

$$\Theta^C = \{D, R, W, D^F, R^F, W^F\}$$

$$\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}, \theta^{DgF}, \theta^{DbF}, \theta^{RgF}, \theta^{RbF}\}$$

$$A^I = A^D = \{a^D, a^R, a^W, a^{bp}\} \subseteq E_2$$

$$A^{IF} = A^D = \{a^{DF}, a^{RF}, a^{WF}, a^{bpF}\} \subseteq E_2$$

$$A^{Gg} = A^{Rg} = \{a^W\} \subseteq E_2$$

$$A^{Gb} = A^{Rb} = \{a^{bp}\} \subseteq E_2$$

$$A^{GgF} = A^{RgF} = \{a^{WF}\} \subseteq E_2$$

$$A^{GbF} = A^{RbF} = \{a^{bpF}\} \subseteq E_2$$

$$\Pr(\theta^R = \theta^{Rg}) = q$$

$$\Pr(\theta^R = \theta^{Rb}) = 1 - q$$

$$\Pr(\theta^{RF} = \theta^{RgF}) = x$$

$$\Pr(\theta^{RF} = \theta^{RbF}) = 1 - x$$

Again I find perfect Bayesian equilibria of the game.

**Proposition 6** *If the incumbent is a good type, he plays the populist policy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ , whilst he plays the welfaristic policy  $a^W$  if  $q \in (\frac{2}{\beta r(a^D)}(r(a^D) - r(a^W)), 1]$ .*

*Proof:*  $EU(a^D) = r(a^D) + \beta(1 - q)(1 - x)r(a^D)$  is always greater than  $EU(a^R) = r(a^R) + \beta(1 - q)(1 - x)r(a^D)$  and  $EU(a^D)$  is always greater than  $EU(a^{bp}) = r(a^W) + \frac{1}{2}\beta(1 - q)(1 - x)r(a^D)$ .  $EU(a^D)$  is greater than  $EU(a^W)$  for  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ .

**Proposition 7** *If the incumbent is a bad type, he plays the populist policy  $a^D$  if  $q \in [0, \frac{2}{\beta r(a^{bp})(1-x)}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})(1 - x))]$ , whilst he plays the bad policy  $a^{bp}$  if  $q \in (\frac{2}{\beta r(a^{bp})(1-x)}(r(a^D) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})(1 - x)), 1]$ .*

*Proof:*  $EU(a^D) = r(a^D) + \beta(1 - q)(1 - x)r(a^D)$  is always greater than  $EU(a^R) = r(a^R) + \beta(1 - q)(1 - x)r(a^D)$  and  $EU(a^D)$  is always greater than  $EU(a^{bp}) = r(a^W) + \frac{1}{2}\beta(1 - q)(1 - x)r(a^D)$ .  $EU(a^D)$  is greater than  $EU(a^W)$  for  $q \in [0, \frac{2}{\beta r(a^D)}(r(a^D) - r(a^W))]$ .

These results show how the probability to face a foreigner good type directly enters into the equilibrium intervals, meaning that the domestic incumbent changes the policies played not only taking into account the probability to face a good domestic challenger, but also taking into account the probability to find a good foreign Government.

Suppose now to slightly change the previous game, such as the domestic government (good type) plays the populist policy  $a^R$  instead of the welfaristic policy  $a^W$ . The game becomes:

$$\Theta^C = \{D, R, W, D^F, R^F, W^F\}$$

$$\Theta^G = \{\theta^{Dg}, \theta^{Db}, \theta^{Rg}, \theta^{Rb}, \theta^{DgF}, \theta^{DbF}, \theta^{RgF}, \theta^{RbF}\}$$

$$A^I = A^D = \{a^D, a^R, a^W, a^{bp}\} \subseteq E_2$$

$$A^{IF} = A^D = \{a^{DF}, a^{RF}, a^{WF}, a^{bPF}\} \subseteq E_2$$

$$A^{Gg} = A^{Rg} = \{a^R\} \subseteq E_2$$

$$A^{Gb} = A^{Rb} = \{a^{bp}\} \subseteq E_2$$

$$A^{GgF} = A^{RgF} = \{a^{WF}\} \subseteq E_2$$

$$A^{GbF} = A^{RbF} = \{a^{RbF}\} \subseteq E_2$$

$$\Pr(\theta^R = \theta^{Rg}) = q$$

$$\Pr(\theta^R = \theta^{Rb}) = 1 - q$$

$$\Pr(\theta^{RF} = \theta^{RgF}) = x$$

$$\Pr(\theta^{RF} = \theta^{RbF}) = 1 - x$$

**Proposition 8** *If the incumbent is a good type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{r(a^W) + (x\beta - 1)r(a^D)}{\frac{1}{2}\beta r(a^D) - \beta r(a^D)(1-x)}]$ , whilst he plays the welfaristic policy  $a^W$  if  $q \in (\frac{r(a^W) + (x\beta - 1)r(a^D)}{\frac{1}{2}\beta r(a^D) - \beta r(a^D)(1-x)}, 1]$ .*

*Proof:*  $EU(a^D) = r(a^D) + \frac{1}{2}\beta qr(a^D) + \beta(1-q)r(a^D)$  is always greater than  $EU(a^R) = r(a^R) + \frac{1}{2}\beta qr(a^D) + \beta(1-q)r(a^D)$  and  $EU(a^D)$  is always greater than  $EU(a^{bp}) = r(a^{bp}) + \frac{1}{2}\beta(1-q)(1-x)r(a^D)$ .  $EU(a^D)$  is greater than  $EU(a^W)$  for  $q \in [0, \frac{r(a^W) + (x\beta - 1)r(a^D)}{\frac{1}{2}\beta r(a^D) - \beta r(a^D)(1-x)}]$ .

**Proposition 9** *If the incumbent is a bad type, he plays the populist strategy  $a^D$  if  $q \in [0, \frac{(1+\beta)r(a^D) - r(a^{bp}) - \frac{1}{2}\beta r(a^D)(1-x)}{\frac{1}{2}\beta r(a^D)x}]$ , whilst he plays the bad policy  $a^{bp}$  if  $q \in (\frac{(1+\beta)r(a^D) - r(a^{bp}) - \frac{1}{2}\beta r(a^D)(1-x)}{\frac{1}{2}\beta r(a^D)x}, 1]$ .*

*Proof:*  $EU(a^D) = r(a^D) + \beta qr(a^D) + \beta(1-q)r(a^D)$  is always greater than  $EU(a^R) = r(a^R) + \beta qr(a^D) + \beta(1-q)r(a^D)$  and  $EU(a^D)$  is always greater than  $EU(a^{bp}) = r(a^{bp}) + \frac{1}{2}\beta(1-q)(1-x)r(a^D)$ .  $EU(a^D)$  is greater than  $EU(a^W)$  for  $q \in [0, \frac{(1+\beta)r(a^D) - r(a^{bp}) - \frac{1}{2}\beta r(a^D)(1-x)}{\frac{1}{2}\beta r(a^D)x}]$ .

Table 1 compares equilibria obtained in the absence of yardstick competition case with equilibria obtained in the presence of the yardstick competition. It is easy to see that strategies played by governments do not differ in the two cases. Otherwise, the difference refers to the broadness of equilibria intervals. In the yardstick competition case intervals where the bad government adopts a mimicking strategy are narrower than those obtained

in the absence of yardstick competition case, which suggests that separating equilibria are more likely to happen. Furthermore, this suggests the important role played by the yardstick competition as a *self-selection device*. Once voters have the possibility to compare domestic and foreign policies, bad candidates realize that it becomes more costly to mimic the good governments and then prefer to immediately separate. This is a positive result because it shows as bad candidates are more likely to be recognized and thrown out of the office. Otherwise, the yardstick competition fails to achieve the *discipline device* goal. In fact, bad politicians are not forced to behave efficiently and thus this entails that voters are worse off. In this economic environment, the conclusion about the role of the yardstick competition is that there exists a trade-off between the *self-selection* goal and the *discipline device* goal. Since I supposed the existence of perfect correlation among economies, my results support the theory by Bordignon et al. ([7]) which affirms that “the larger is the degree of correlation between economies, the more the citizen learns by observing the fiscal choices in the other jurisdiction, and the more difficult it is for the bad politician to escape detection when cheating ”.

environment	eq.strategy/intervals
1/n/g	$a^D / [0, \frac{2}{\beta r(a^D)} (r(a^D) - r(a^W))]$ $a^W / (\frac{2}{\beta r(a^D)} (r(a^D) - r(a^W)), 1]$
1/y/g	$a^D / [0, \frac{2}{\beta r(a^D)} (r(a^D) - r(a^W))]$ $a^W / (\frac{2}{\beta r(a^D)} (r(a^D) - r(a^W)), 1]$
1/n/b	$a^D / [0, \frac{2}{\beta r(a^{bp})} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp}))]$ $a^{bp} / (\frac{2}{\beta r(a^{bp})} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp})), 1]$
1/y/b	$a^D / [0, \frac{2}{\beta r(a^{bp})(1-x)} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp})(1-x))]$ $a^{bp} / (\frac{2}{\beta r(a^{bp})(1-x)} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp})(1-x)), 1]$
2/n/g	$a^D / [0, \frac{2}{\beta r(a^D)} (r(a^D) - r(a^W))]$ $a^W / (\frac{2}{\beta r(a^D)} (r(a^D) - r(a^W)), 1]$
2/y/g	$a^D / [0, \frac{-r(a^D) + r(a^W) + x \beta r(a^D)}{\frac{1}{2} \beta r(a^D) - \beta r(a^D)(1-x)}]$ $a^W / (\frac{-r(a^D) + r(a^W) + x \beta r(a^D)}{\frac{1}{2} \beta r(a^D) - \beta r(a^D)(1-x)}, 1]$
2/n/b	$a^D / [0, \frac{2}{\beta r(a^{bp})} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp}))]$ $a^{bp} / (\frac{2}{\beta r(a^{bp})} (r(a^D) - r(a^{bp}) + \frac{1}{2} \beta r(a^{bp})), 1]$
2/y/b	$a^D / [0, \frac{(1+\beta)r(a^D) - r(a^{bp}) - \frac{1}{2} \beta r(a^D)(1-x)}{\frac{1}{2} \beta r(a^D)x}]$ $a^{bp} / (\frac{(1+\beta)r(a^D) - r(a^{bp}) - \frac{1}{2} \beta r(a^D)(1-x)}{\frac{1}{2} \beta r(a^D)x}, 1]$

**Table 1** - equilibrium strategy - *Legend*: (1) refers to the case where the good government is committed to play the welfaristic strategy and (2) where is committed to play the populist strategy; (n) indicitates the absence of yardstick competition case and (y) the presence of yardstick competition case and (g) indicates the government good type and (b) the government bad type.

## 6 A game with incomplete information and presence of shocks

In this second part, I allow for the possibility that some shocks may occur in the economy; these shocks may be seen as all of those exogenous events which may increase (or decrease) efficiency in the production of public goods. Shocks may be either positive (P) or negative (N); if a shock is positive, then the efficiency in the production increases whilst if the shock is negative the efficiency decreases. An increase in efficiency may be seen as the possibility to produce the same amount of good at a lower cost, or to produce an higher amount of that good at the same cost. The cost of production is borne by taxation and thus, citizens are better off when a shock is positive, since they pay less taxes. An important assumption here is that the sign and the magnitude of the shock is a private information of candidates, which perfectly observe whether these are positive or negative. Otherwise, voters only perceive the existence of shocks but are not able to measure neither the magnitude nor the sign. Thus, at the beginning of the game, Nature choose both the type of candidates and the type of shock. Candidates observe Nature's choice and then announce (and commit) to a policy which depends on the type of shock, that is  $a(\epsilon)$ ; voters observe candidate's policies and vote for the candidate who chooses the nearest policy with respect to their IP.

### 6.1 Positive shock

Suppose at first that the economy is affected by a positive shock. I write the preferences for every government type.

$$\begin{aligned}\theta^{Dg} : r(a^D(\epsilon^P)) &> r(a^W(\epsilon^P)) > r(a^R(\epsilon^P)) > 0 > r(a^D(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^R(\epsilon^N)) > r(a^{bp}) \\ \theta^{Db} : r(a^{bp}) &> r(a^D(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^R(\epsilon^N)) > 0 > r(a^D(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^R(\epsilon^P)) \\ \theta^{Rg} : r(a^R(\epsilon^P)) &> r(a^W(\epsilon^P)) > r(a^D(\epsilon^P)) > 0 > r(a^R(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^D(\epsilon^N)) > r(a^{bp}) \\ \theta^{Rb} : r(a^{bp}) &> r(a^R(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^D(\epsilon^N)) > 0 > r(a^R(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^D(\epsilon^P))\end{aligned}$$

[FIGURE 5 HERE]

Figure 5 shows the IP of each government type when the shock is positive. Line BC indicates the locus where the budget constraint is equal to zero. This is a private information of the government. Notice that the welfaristic strategy  $W$  and the strategy played by government good type  $GDg$  and  $GRg$  stands exactly over the budget constraint line, while those played by government bad type entails tax rates which are higher than those needed to clear the budget. This means that bad government collect more taxes and the difference between the taxes collected and taxes needed to clear the budget represents the government's rent. Graphically, this rent is represented by the distance between the Government bad type IP and the budget constraint line.



**Proposition 10** *If the incumbent is a good type, he plays the populist strategy  $a^D(\epsilon^P)$  if  $q \in [0, \frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P)))]$ , whilst he plays the welfaristic policy  $a^W(\epsilon^P)$  if  $q \in (\frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P))), 1]$ .*

*Proof:* The bad policy, policies which are a function of the negative shock and policies which stand on the B's IP are strictly dominated by  $a^D(\epsilon^P)$  and  $a^W(\epsilon^P)$ .

Notice here that a government (good type) always chooses a policy which stands over the budget constraint line, even though the exact location over this line  $a^D(\epsilon^P)$  or  $a^W(\epsilon^P)$  changes with respect to probability intervals.

**Proposition 11** *If the incumbent is a bad type, he plays the populist strategy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{2}{\beta r(a^{bp}(\epsilon^P))}(r(a^D(\epsilon^N)) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp}))]$ , whilst he plays the bad policy  $a^{bp}$  if  $q \in (\frac{2}{\beta r(a^{bp}(\epsilon^P))}(r(a^D(\epsilon^N)) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})), 1]$*

*Proof:* the bad policy and the populist strategy strictly dominates all the other strategies.

Notice in this case how a bad incumbent is willing to make voters believe that a shock is negative, since if they believed so, he would be able to adopt a policy which would stand at a closer point to his IP.

## 6.2 Negative shock

Suppose now that the economy is affected by a negative shock and write again the preferences for every government type.

$$\theta^{Dg} : r(a^D(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^R(\epsilon^N)) > 0 > r(a^D(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^R(\epsilon^P)) > r(a^{bp})$$

$$\theta^{Db} : r(a^{bp}) > r(a^D(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^R(\epsilon^N)) > 0 > r(a^D(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^R(\epsilon^P))$$

$$\theta^{Rg} : r(a^R(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^D(\epsilon^N)) > 0 > r(a^R(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^D(\epsilon^P)) > r(a^{bp})$$

$$\theta^{Rb} : r(a^{bp}) > r(a^R(\epsilon^N)) > r(a^W(\epsilon^N)) > r(a^D(\epsilon^N)) > 0 > r(a^R(\epsilon^P)) > r(a^W(\epsilon^P)) > r(a^D(\epsilon^P))$$

[FIGURE 6 HERE] Figure 6 shows the Ideal Point of each government type when the shock is negative. Line BC indicates the locus where the budget constraint is equal to zero. Unlike the previous case, this time the budget constraint stand north-eastward; this means that is more costly for Government to produce public goods and it needs a higher level of taxation. Notice also how the welfaristic strategy  $W$  does not stand over the same line where the strategy played by government good type  $GDg$  and  $GRg$  stands. Indeed, every time taxes are lower, welfare of welfaristic voters increases and then, they would accept that the government overspend its budget if this would help to increase his wealth. As before, strategies played by government bad type entail higher tax rates than those needed to clear the budget.

**Proposition 12** *If the incumbent is a good type, he plays the populist policy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{2}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N))))$ , whilst he plays the welfaristic policy  $a^W(\epsilon^P)$  if  $q \in (\frac{2}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N))), 1]$ .*

*Proof:* same as for the positive shock.

**Proposition 13** *If the incumbent is a bad type, he plays the bad policy  $a^{bp}$  if  $q \in [0, \frac{2}{\beta r(a^{bp})}(r(a^{bp}) - r(a^R(\epsilon^N)) - \frac{1}{2}\beta r(a^{bp})))$ , whilst he plays the populist policy  $a^R(\epsilon^P)$  if  $q \in (\frac{2}{\beta r(a^{bp})}(r(a^{bp}) - r(a^R(\epsilon^N)) - \frac{1}{2}\beta r(a^{bp})), 1]$ .*

*Proof:* same as for the positive shock.

## 7 A case of yardstick competition with presence of shocks

I analyse now the case of yardstick competition in the presence of shocks where, once again, shocks may be either positive or negative.

### 7.1 The shock is positive

I suppose that the domestic good incumbent's preferred policy is  $a^R(\epsilon^P)$ , whilst the bad incumbent's preferred policy is  $a^{bp}$ . Furthermore, the foreign good incumbent's preferred policy is  $a^W(\epsilon^P)$ .

**Proposition 14** *If the incumbent is a good type, he chooses the populist policy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{1}{\beta r(a^D(\epsilon^P))(1-x)}(r(a^D(\epsilon^P))(x\beta - 1) + r(a^W(\epsilon^P)))]$ , whilst he chooses the welfaristic policy  $a^W(\epsilon^P)$  if  $q \in (\frac{1}{\beta r(a^D(\epsilon^P))(1-x)}(r(a^D(\epsilon^P))(x\beta - 1) + r(a^W(\epsilon^P))), 1]$ .*

*Proof:* same as for the absence of yardstick competition case.

**Proposition 15** *If the incumbent is a bad type, he chooses the populist policy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x)))]$ , whilst he chooses the bad policy  $a^{bp}$  if  $q \in (\frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x))), 1]$ .*

*Proof:* same as for the absence of yardstick competition case.

### 7.2 The shock is negative

**Proposition 16** *If the incumbent is a good type, he chooses the populist policy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{1}{\beta r(a^D(\epsilon^N))}(r(a^W(\epsilon^N)) - r(a^D(\epsilon^N)))]$ , whilst he chooses the welfaristic policy  $a^W(\epsilon^P)$  if  $q \in (\frac{1}{\beta r(a^D(\epsilon^N))}(r(a^W(\epsilon^N)) - r(a^D(\epsilon^N))), 1]$ .*

*Proof:* same as for the absence of yardstick competition case.

**Proposition 17** *If the incumbent is a bad type, he chooses the populist policy  $a^D(\epsilon^N)$  if  $q \in [0, \frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x)))]$ , whilst he chooses the bad policy  $a^{bp}(\epsilon^P)$  if  $q \in (\frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x))), 1]$*

*Proof:* same as for the absence of yardstick competition case.

Table 2 summarizes overall results for an economy characterized by shocks.

environment	eq.strategy/intervals
1/n/g/p	$a^D(\epsilon^P)/[0, \frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P)))]$ $a^W(\epsilon^P)/(\frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P))), 1]$
1/n/g/n	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)))]$ $a^W(\epsilon^N)/(\frac{2}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N))), 1]$
1/n/b/p	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})}(r(a^D(\epsilon^N)) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp}))]$ $a^{bp}/(\frac{2}{\beta r(a^{bp})}(r(a^D(\epsilon^N)) - r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})), 1]$
1/n/b/n	$a^D(\epsilon^N)/[0, \frac{2(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)))}{\beta r(a^{bp})}] \text{ if } \beta > \frac{2(r(a^{bp}) - r(a^A(\epsilon^N)))}{r(a^{bp})}$ $a^W(\epsilon^N)/(\frac{2(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)))}{\beta r(a^{bp})}), 1]$ $a^{bp}/[0, \frac{2(r(a^{bp})(1 - \frac{\beta}{2}) + r(a^W(\epsilon^N)))}{\beta r(a^{bp})}] \text{ if } \beta < \frac{2(r(a^{bp}) - r(a^A(\epsilon^N)))}{r(a^{bp})}$ $a^W(\epsilon^N)/(\frac{2(r(a^{bp})(1 - \frac{\beta}{2}) + r(a^W(\epsilon^N)))}{\beta r(a^{bp})}), 1]$
1/y/g/p	$a^D(\epsilon^P)/[0, \frac{1}{\beta r(a^D(\epsilon^P))(1-x)}(r(a^D(\epsilon^P))(x\beta - 1) + r(a^W(\epsilon^P)))]$ $a^W(\epsilon^P)/(\frac{1}{\beta r(a^D(\epsilon^P))(1-x)}(r(a^D(\epsilon^P))(x\beta - 1) + r(a^W(\epsilon^P))), 1]$
1/y/g/n	$a^D(\epsilon^N)/[0, \frac{1}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)))]$ $a^W(\epsilon^N)/(\frac{1}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N))), 1]$
1/y/b/p	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x)))]$ $a^{bp}/(\frac{2}{\beta r(a^{bp})(1-x)}(r(a^D(\epsilon^N)) - r(a^{bp})(1 - \frac{\beta}{2}(1-x))), 1]$
1/y/b/n	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})}(\frac{1}{2}\beta r(a^{bp}) - r(a^{bp}) + r(a^D(\epsilon^N)))]$ $a^{bp}/(*, \frac{2}{\beta r(a^{bp})}(-\frac{1}{2}\beta r(a^{bp}) + r(a^{bp}) - r(a^W(\epsilon^N))))$ $a^W(\epsilon^N)/(\frac{2}{\beta r(a^{bp})}(-\frac{1}{2}\beta r(a^{bp}) + r(a^{bp}) - r(a^W(\epsilon^N))), 1]$
2/n/g/p	$a^D(\epsilon^P)/[0, \frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) + \frac{1}{2}\beta r(a^D(\epsilon^P)) - r(a^W(\epsilon^P)))]$ $a^W(\epsilon^P)/(\frac{2}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) + \frac{1}{2}\beta r(a^D(\epsilon^P)) - r(a^W(\epsilon^P))), 1]$
2/n/g/n	$a^D(\epsilon^N)/[0, \frac{1}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N))(1 + \frac{\beta}{2}) - r(a^W(\epsilon^N)))]$ $a^W(\epsilon^N)/(\frac{1}{\beta r(a^D(\epsilon^N))}(r(a^D(\epsilon^N))(1 + \frac{\beta}{2}) - r(a^W(\epsilon^N))), 1]$
2/n/b/p	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})}(r(a^{bp})(\frac{\beta}{2} - 1) + r(a^D(\epsilon^N)))]$ $a^{bp}/(\frac{2}{\beta r(a^{bp})}(r(a^{bp})(\frac{\beta}{2} - 1) + r(a^D(\epsilon^N))), 1]$
2/n/b/n	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})}((\frac{1}{2}\beta - 1)r(a^{bp}) + r(a^D(\epsilon^N)))]$ $a^{bp}/(*, \frac{1}{\beta r(a^{bp})}((\frac{1}{2}\beta + 1)r(a^{bp}) - r(a^W(\epsilon^N))))$ $a^W(\epsilon^N)/(\frac{1}{\beta r(a^{bp})}((12\beta + 1)r(a^{bp}) - r(a^W(\epsilon^N))), 1]$
2/y/g/p	$a^D(\epsilon^P)/[0, \frac{1}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P)) + \frac{1}{2}\beta r(a^D(\epsilon^P)))]$ $a^W(\epsilon^P)/(\frac{1}{\beta r(a^D(\epsilon^P))}(r(a^D(\epsilon^P)) - r(a^W(\epsilon^P)) + \frac{1}{2}\beta r(a^D(\epsilon^P))), 1]$
2/y/g/n	$a^D(\epsilon^N)/[0, \frac{1}{\beta r(a^D(\epsilon^N))(1-x)}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)) + \frac{1}{2}\beta r(a^D(\epsilon^N))(1-x))]$ $a^W(\epsilon^N)/(\frac{1}{\beta r(a^D(\epsilon^N))(1-x)}(r(a^D(\epsilon^N)) - r(a^W(\epsilon^N)) + \frac{1}{2}\beta r(a^D(\epsilon^N))(1-x)), 1]$
2/y/b/p	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})(1-x)}(a^D(\epsilon^N) + \frac{1}{2}\beta r(a^{bp})(1-x) - r(a^{bp}))]$ $a^{bp}/(\frac{2}{\beta r(a^{bp})(1-x)}(a^D(\epsilon^N) + \frac{1}{2}\beta r(a^{bp})(1-x) - r(a^{bp})), 1]$
2/y/b/n	$a^D(\epsilon^N)/[0, \frac{2}{\beta r(a^{bp})(1-x)}(\frac{1}{2}\beta r(a^{bp})(1-x) - r(a^{bp}) + r(a^D(\epsilon^N)))]$ $a^{bp}/(*, \frac{2}{\beta r(a^{bp})}(-r(a^W(\epsilon^N)) + r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})(1-x)))$ $a^W(\epsilon^N)/(\frac{2}{\beta r(a^{bp})(2-x)}(-r(a^W(\epsilon^N)) + r(a^{bp}) + \frac{1}{2}\beta r(a^{bp})(1-x)), 1]$

**Table 2** - equilibrium strategy - *Legend:* (1) refers to the case where the good government is committed to play the welfaristic strategy and (2) where is committed to play the populist strategy; (n) indicates the absence of yardstick competition case and (y) the presence of yardstick competition case and (g) indicates the government good type and (b) the government bad type. (\*)  $\frac{2}{\beta r(a^{bp})}(\frac{1}{2}\beta r(a^{bp}) - r(a^{bp}) + r(a^D(\epsilon^N)))$ ; (\*\*)  $\frac{2}{\beta r(a^{bp})}(\frac{1}{2}\beta r(a^{bp}) - r(a^{bp}) + r(a^D(\epsilon^N)))$ ; (\*\*\*)  $\frac{2}{\beta r(a^{bp})(1-x)}(\frac{1}{2}\beta r(a^{bp})(1-x) - r(a^{bp}) + r(a^D(\epsilon^N)))$

The comparison of results in Table 2 enables us to evaluate the effect of the yardstick competition both as a selective and discipline device. The first observation is that values assumed by parameters of the model (the probability that the foreign government is good and the discount rate) are crucial. Thus, to understand how the introduction of the yardstick competition affects the equilibria, I performed some numerical simulations. Table 3 shows the payoffs for every strategy played and for each type and shock (i.e. b/p stands for bad player and positive shock). Furthermore, Table 4 shows the numerical values which intervals assumes for every equilibrium strategy played in every situation. First of all, notice that when the type is good and the shock is positive the interval where the strategy  $a^D(\epsilon^P)$  ( $a^W(\epsilon^P)$ ) is played is narrower (broader) in the absence of the yardstick competition, for every value of  $\beta$  and  $x$ . This means that the yardstick competition forces the good government to play the welfaristic strategy. In this case, it is possible to argue that the introduction of another government acts as a discipline device for the domestic government behavior. The same holds when the shock is negative. When the type is bad, we must distinguish two cases. The first case is when the shock is positive; we are in a situation where the interval where the bad government plays the bad policy is broader than the yardstick competition case, meaning that the existence of a benchmark government forces the bad incumbent to separate in the first period and lose elections. This is a case where the yardstick competition acts a selection device. The second case is when the shock is negative. In this situation we must distinguish other two sub-cases, which depend on the parameter  $\beta$ . In fact, when the discount rate is sufficiently high ( $\beta > 0.2$ ) the bad incumbent never plays the bad policy, whilst when  $\beta$  is sufficiently low ( $\beta < 0.2$ ) the bad incumbent replaces the populist policy previously played, with the bad policy. This is obvious. When the incumbent discount future more, it prefers to immediately gain the first-period rent, whilst if it discounts future less, it prefers mimicking the good government in the first period to win elections and playing the bad policy only at the second period. The introduction of the yardstick competition produces more difficult results to interpret. In fact, we see that the incumbent may play three equilibrium strategies depending on the parameters of the model.

	g/p	b/p	g/n	b/n
$r(a^D(\epsilon^P))$	1	0.6	0.7	0.6
$r(a^W(\epsilon^P))$	0.9	0.5	0.6	0.5
$r(a^R(\epsilon^P))$	0.8	0.4	0.5	0.4
$r(a^D(\epsilon^N))$	0.7	0.9	1	0.9
$r(a^W(\epsilon^N))$	0.6	0.8	0.9	0.8
$r(a^R(\epsilon^N))$	0.5	0.7	0.8	0.7
$r(a^{bp})$	0.4	1	0.4	1

**Table 3** - Numerical Simulations -

environment	eq.strategy/intervals
1/n/g/p	$a^D(\epsilon^P)/[0, \frac{0.2}{\beta})$ $a^W(\epsilon^P)/(\frac{0.2}{\beta}, 1]$
1/n/g/n	$a^D(\epsilon^N)/[0, \frac{0.2}{\beta})$ $a^W(\epsilon^N)/(\frac{0.2}{\beta}, 1]$
1/n/b/p	$a^D(\epsilon^N)/[0, \frac{0.2(0.5\beta-0.1)}{\beta})$ $a^{bp}/(\frac{0.2(0.5\beta-0.1)}{\beta}, 1]$
1/n/b/n	$a^D(\epsilon^N)/[0, \frac{0.2}{\beta})$ if $\beta > 0.2$ $a^W(\epsilon^N)/(\frac{0.2}{\beta}, 1]$ $a^{bp}/[0, \frac{0.2(1.8-0.5\beta)}{\beta})$ if $\beta < 0.2$ $a^W(\epsilon^N)/(\frac{0.2(1.8-0.5\beta)}{\beta}, 1]$
1/y/g/p	$a^D(\epsilon^P)/[0, \frac{(x\beta-0.1)}{\beta(1-x)})$ $a^W(\epsilon^P)/(\frac{(x\beta-0.1)}{\beta(1-x)}, 1]$
1/y/g/n	$a^D(\epsilon^N)/[0, \frac{0.1}{\beta})$ $a^W(\epsilon^N)/(\frac{0.1}{\beta}, 1]$
1/y/b/p	$a^D(\epsilon^N)/[0, 0.2 \frac{(0.5\beta(1-x)-0.1)}{\beta(1-x)})$ $a^{bp}/(0.2 \frac{(0.5\beta(1-x)-0.1)}{\beta(1-x)}, 1]$
1/y/b/n	$a^D(\epsilon^N)/[0, 2 \frac{(0.5\beta-0.1)}{\beta})$ $a^{bp}/(2 \frac{(0.5\beta-0.1)}{\beta}, 2 \frac{(0.5\beta+0.2)}{\beta})$ $a^W(\epsilon^N)/(2 \frac{(0.5\beta+0.2)}{\beta}, 1]$
2/n/g/p	$a^D(\epsilon^P)/[0, 2 \frac{(0.1+0.5\beta)}{\beta})$ $a^W(\epsilon^P)/(2 \frac{(0.1+0.5\beta)}{\beta}, 1]$
2/n/g/n	$a^D(\epsilon^N)/[0, \frac{(0.1+0.5\beta)}{\beta})$ $a^W(\epsilon^N)/(\frac{(0.1+0.5\beta)}{\beta}, 1]$
2/n/b/p	$a^D(\epsilon^N)/[0, 2 \frac{(0.5\beta-0.1)}{\beta})$ $a^{bp}/(2 \frac{(0.5\beta-0.1)}{\beta}, 1]$
2/n/b/n	$a^D(\epsilon^N)/[0, 2 \frac{(0.5\beta-0.1)}{\beta})$ $a^{bp}/(2 \frac{(0.5\beta-0.1)}{\beta}, 2 \frac{(0.5\beta+0.2)}{\beta})$ $a^W(\epsilon^N)/(2 \frac{(0.5\beta+0.2)}{\beta}, 1]$
2/y/g/p	$a^D(\epsilon^P)/[0, 2 \frac{(0.5\beta+0.1)}{\beta})$ $a^W(\epsilon^P)/(2 \frac{(0.5\beta+0.1)}{\beta}, 1]$
2/y/g/n	$a^D(\epsilon^N)/[0, \frac{(0.5\beta(1-x)+0.1)}{\beta(1-x)})$ $a^W(\epsilon^N)/(\frac{(0.5\beta(1-x)+0.1)}{\beta(1-x)}, 1]$
2/y/b/p	$a^D(\epsilon^N)/[0, \frac{2(0.5\beta(1-x)-0.1)}{\beta(1-x)})$ $a^{bp}/(\frac{2(0.5\beta(1-x)-0.1)}{\beta(1-x)}, 1]$
2/y/b/n	$a^D(\epsilon^N)/[0, \frac{2(0.5\beta(1-x)-0.1)}{\beta(1-x)})$ $a^{bp}/(\frac{2(0.5\beta(1-x)-0.1)}{\beta(1-x)}, \frac{2(0.5\beta(1-x)+0.2)}{\beta(1-x)})$ $a^W(\epsilon^N)/(\frac{2(0.5\beta(1-x)+0.2)}{\beta(1-x)}, 1]$

**Table 4** - Numerical Simulations -

## 8 Conclusion

In this papers I analysed a typical yardstick competition game using a spatial voting model, where voters vote for a candidate according to the distance between their IP and the policy selected by a candidate. The policy which is closer to a voter's IP provides the voter with a higher utility so that minimizing the distance means maximising the utility. I analysed a benchmark case with complete information and then I studied the effects of the asymmetrical information between candidates and voters. I demonstrated that, in this framework, a yardstick competition entails a selection but not a discipline device, suggesting the existence of a trade off between the two goals. In the second part, I analysed an economy characterised by the presence of shocks, whose sign and magnitude are a private information of incumbents. This time, the introduction of the yardstick competition acts both as a selection and a discipline device. Of course, this is a real optimistic achievement and suggests that providing information to voters about the tax rates chosen by local governments would entail an increase of discipline among politicians. Unfortunately, hardly in the real world one may observe that voters are completely informed about policies' outcome. Furthermore, this model consider an electorate who votes without any concern about ideological or idiosyncratic preferences. This open more than an opportunity of research for future works.

## 9 Appendix A

Suppose  $A$  be a positive definite matrix of weights,  $x$  the preferred position (or ideal point, IP) of a voter  $i$  and  $a$  the policy vector chosen by a candidate, with  $a \in E_2$ . Then  $\|a - x\|$  represents a quadratic metric loss function or, in other words, the loss which a voter suffers for not to stand on his IP<sup>5</sup>.

Furthermore, define a representative voter's utility function as  $u_i(\|a - x\|_A)$ , where  $\frac{\partial u_i}{\partial \|a - x\|_A} < 0$ , with  $(\|a - x\|_A)^2 = (a - x)'A(a - x)$

**Lemma 1** *Suppose there are two policies  $a'$  and  $a''$ . We say that  $a'$  is preferred to  $a''$  and we denote it with the expression  $a'Pa''$  if and only if  $\|a' - x\| < \|a'' - x\|$  and thus  $u_i(\|a' - x\|) < u_i(\|a'' - x\|)$ . Otherwise, we say that  $a''$  is preferred to  $a'$  and we denote it with the expression  $a''Pa'$  if and only if  $\|a'' - x\| < \|a' - x\|$  and thus  $u_i(\|a'' - x\|) < u_i(\|a' - x\|)$ . Finally, we say that  $a'$  is indifferent to  $a''$  and we denote it with the expression  $a'Ia''$  if and only if  $\|a' - x\| = \|a'' - x\|$  and thus  $u_i(\|a' - x\|) = u_i(\|a'' - x\|)$ .*

Let us now introduce the event *voter  $i$  votes for  $a^*$*  and denote the probability of the event with  $\Pr(i \text{ votes for } a')$  with  $\Pr_i(\|a' - x\| - \|a'' - x\|)$  and the probability of the event  $\Pr(i \text{ votes for } a'')$  with  $1 - \Pr_i(i \text{ votes for } a'')$ , where  $\Pr$  is a monotonically non decreasing function, differentiable and non constant (i.e.  $\Pr' > 0$ )

**Lemma 2** *for every  $a' \in E_2$  and  $a'' \in E_2$  we say that  $a'Ra''$  but that not  $a''Ra'$  if and only if  $\Pr_i(\|a' - x\| \leq \|a'' - x\|) < \frac{1}{2}$ . This is called the majority rule. Otherwise, if  $a'Ra''$  and  $a'Ra''$  it must be  $a'Ia''$ .*

Note that when  $|a' - x| = |a'' - x|$  then  $\Pr(0) = \frac{1}{2}$ .  
In a probabilistic voting model, the probability of voting the policy  $a'$  is a function of the single voter's IP. That is  $y = \frac{a' + a''}{\alpha}$  where  $y$  is a random variable must be:

$$\Pr_i(y) = 1 \text{ if } y > \delta$$

$$\Pr_i(y) = \frac{1}{2} \text{ if } y = \delta$$

$$\Pr_i(y) = 0 \text{ if } y < \delta$$

Furthermore, let us introduce the following definitions which deal with the concept of *dominance* and *transitivity*.

**Lemma 3** *We say that a policy  $a^*$  is dominant if and only if  $a^*Ra$ , for every  $a \in E_2$*

Thus, a necessary and sufficient condition is that a policy  $a^*$  is dominant if and only if, for every point  $t \in E_2$  and every number  $b > 0$ , it follows that  $\Pr[t'(X - a^* \leq b)] \geq \frac{1}{2}$ . A natural problem here arises, due to the multidimensionality of space: indeed it may be the case where a dominant point may not exist. It can be demonstrated that if the probability density function (or the discrete frequency function)  $P^*$  is symmetric about  $a^*$ , then  $a^*$  is dominant. Examples of distribution functions which are symmetric about some point  $a^*$ :

- a discrete distribution on a set of  $2k + 1$  points in  $E_n \{0, a_1, -a_1, \dots, a_k, -a_k\}$ , such that  $f(a_i) = -f(a_i)$ , for  $i = 1, \dots, k$ . With this type of distribution  $a^* = 0$ .
- a multivariate normal distribution with mean  $\mu$  and non-singular covariance matrix  $\Sigma$ . With this type of distribution  $a^* = \mu$ .
- the probability density  $f$  on  $E_2$  defined by  $f(a) = \frac{1}{2}[f_1(a) + f_2(a)]$ , where  $f_1$  is a multivariate normal density with mean  $\mu_1$  and non-singular covariance matrix  $\Sigma$  and  $f_2$  is a multivariate normal density with mean  $\mu_2$  and non-singular covariance matrix  $\Sigma$ . For this distribution  $a^* = \frac{1}{2}(\mu_1 + \mu_2)$ .

---

<sup>5</sup>Notice that, if the voter stands on his ideal point,  $x = a$  and the loss function is equal to zero



**Lemma 4** *The distribution  $P^*$  is said to have a unique median in all directions if, for every policy  $a \in E_2$  with  $a \neq 0$  there is a unique real number  $b$  such that both  $\Pr(a'X \leq b) \geq \frac{1}{2}$  and  $\Pr(a'X \geq b) \geq \frac{1}{2}$ . Indeed if  $P^*$  has a unique median in all directions, and supposing that there exists a dominant policy  $a^*$ , then for any two policies  $a' \in E_2$  and  $a'' \in E_2$ ,  $\|a' - a^*\| < \|a'' - a^*\|$  if and only if  $a'Pa''$ .*

The consequences of Lemma 4 are twofold. First of all, the relation  $R$  is transitive and completely orders all of the points  $E_2$ . Secondly it says that if policy  $a^*$  exists, then it is unique.

## 10 Appendix B

In this Appendix I provide a demonstration of a game with complete information in a *multivariate* case. Suppose that  $x$  is a vector and  $f(x)$  a multivariate density function. The vote for a candidate can be expressed in vector notation as:

$$V(a', a'') = \int_R f(X)g(x - a)dx$$

where,

$$R = \{x \mid \|x - a'\| < \|x - a''\|\} \subset E_n$$

where  $R$  is a set in a  $n$ -dimensional Euclidean space which contains the most preferred positions of all the voters who prefer  $a'$  to  $a''$ . Assume now that  $y = x - a$  and  $\xi = a' - a''$ , so that  $\|y\| = \|x - a\|$  and  $\|y + \xi\| = \|a' - a''\|$ . Thus,  $R = \{y : \|y\| < \|y + \xi\|\}$ , so that we get

$$V(a', a'') = \int_R f(y + a)g(y)dy$$

Rearrange  $R$ , we obtain:

$$\begin{aligned} R &= \{y : \|y\| < \|y + \xi\|\} = \{y : (y_1)^2 + (y_2)^2 < (y_1 - \xi_1)^2 - (y_2 - \xi_2)^2\} \\ &= \{y : 0 < 2\xi_1 y_1 + 2\xi_2 y_2\} \end{aligned}$$

Define  $y^* = -\frac{2\xi_2 y_2 + (\xi_1)^2 + (\xi_2)^2}{2\xi_1}$  we obtain  $R = \{y : y_1 > y^*\}$ . Without loss of generality, we assume that one of the policy is greater than the other; assume, for instance that  $a' < a''$ . We use the expression of  $\xi = a' - a''$  so that we can express the vector notation as

$$V(a', a'') = \int_{-\infty}^{\infty} \int_{-\infty}^{y^*} f(y_1 + a'_1, y_2 + a'_2)g(y_1, y_2)dy_1 dy_2$$

Deriving  $V(a', a'')$  with respect to  $a'_1$ , we obtain

$$\frac{\partial y^*}{\partial a'_1} = \frac{2\xi_2 y_2 + (\xi_2)^2}{(\xi_1)^2} - \frac{1}{2}$$

By using the Leibnitz's rule we obtain:

$$\frac{\partial V(a', a'')}{\partial a'_1} = \int_R \frac{\partial f(y + a)}{\partial x_1} g(y)dy + \int_{-\infty}^{+\infty} f(y^* + a'_1, y_2 + a'_2)g(y^*, y_2) \left[ \frac{2\xi_2 y_2 + (\xi_2)^2}{(\xi_1)^2} - \frac{1}{2} \right] dy_2$$

the surprising result we achieve is the existence of an equilibrium where candidates' policies do not converge; otherwise, there exists an asymmetric and opposite position with respect to the mean on either the major or minor axis of  $f(x)$ , let  $a'_1 = -a'_1$  and  $a'_2 = a'_2 = 0$ . Thus, we obtain that  $\xi_1 = 2a'_1$ ,  $\xi_2 = 0$  and  $y_1^* = a'_1$ .

$$\frac{\partial V(a', a'')}{\partial a'_1} = \int_{-\infty}^{+\infty} \int_{-\infty}^0 \frac{\partial f(x_1, x_2)}{\partial x_1} g(x_1 - a'_1, x_2)dx_1 dx_2 - \frac{1}{2} \int_{-\infty}^{+\infty} f(0, x_2)g(-a'_1, x_2)dx_2$$

We can easily see that  $\frac{\partial V(a', a'')}{\partial a'_1}$  can be divided in two components and that an increase in the variance of the first term does not affect the second, which means that an equilibrium where the two candidates are located in different positions may exist and that they do not converge toward each other along either axis of  $f(x)$ . Finally, it can be shown that on an axis of  $f(x)$  at least a local equilibrium exists, where  $\frac{\partial V(a', a'')}{\partial a'_2} = 0$  exists. We observe that  $\frac{\partial y^*}{\partial a'_2} = -\frac{y_2 + \xi_2}{\xi_1}$ . We use again the Leibnitz's rule and obtain:

$$\begin{aligned} \frac{\partial V(a', a'')}{\partial a'_1} &= \int_{-\infty}^{+\infty} \int_{-\infty}^0 \frac{\partial f(x_1, x_2)}{\partial x_2} g(x_1 - a'_1) dy \\ &\quad - \frac{1}{\xi_1} \int_{-\infty}^{+\infty} f(y_1 * + a'_1, y_2 + a'_1) g(y_1, y_2) (y_2 + \xi_2) dy_2 \end{aligned}$$

Imposing  $a'_1 = -a'_1$  and  $a'_2 = a'_2$  as local equilibrium state and  $x = y + a'$  we obtain:

$$\begin{aligned} \frac{\partial V(a', a'')}{\partial a'_1} &= \int_{-\infty}^{+\infty} \int_{-\infty}^0 \frac{\partial f(x_1, x_2)}{\partial x_2} g(x_1 - a'_1, x_2) dx_1 dx_2 \\ &\quad - \frac{1}{2a'_2} \int_{-\infty}^{+\infty} [x_2 - a'_2] f(0, x_2) g(-a'_1, x_2 - a'_2) dx_2 \end{aligned}$$

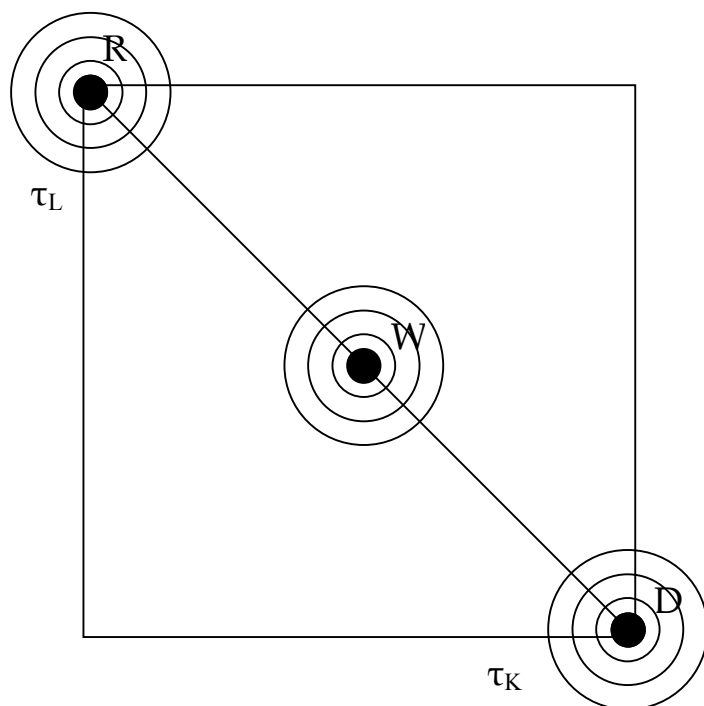
and imposing the condition  $a'_2 = 0$  due to a rotation of the  $f(x)$  axe of the coordinate space we obtain

$$\frac{\partial V(a', a'')}{\partial a'_1} = \int_{-\infty}^{+\infty} \int_{-\infty}^0 x_2 f(0, x_2) g(-a'_1, x_2) dx_2 = 0$$

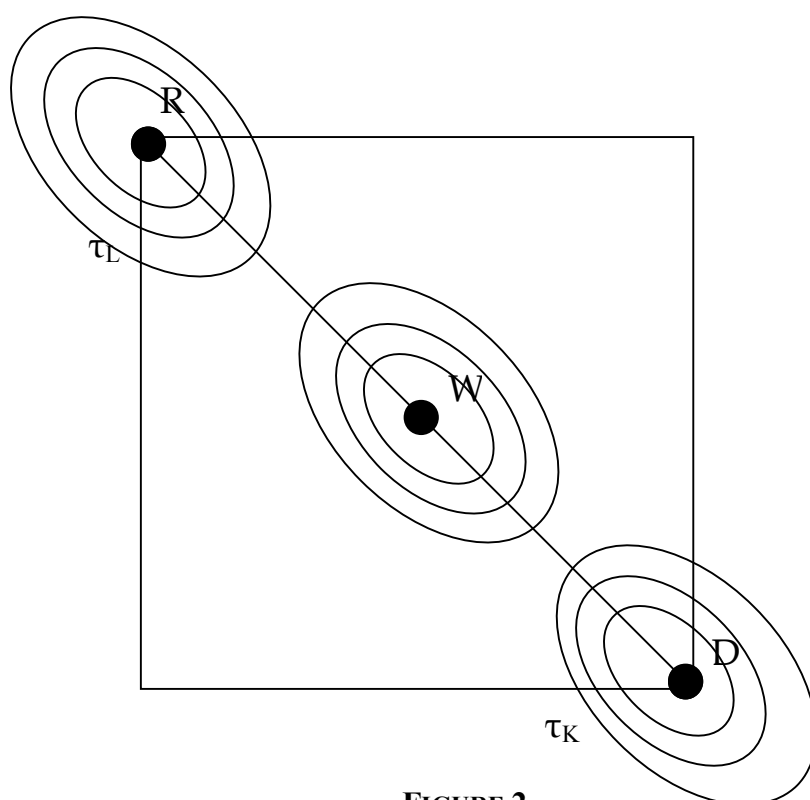
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**FIGURE 1**



**FIGURE 2**

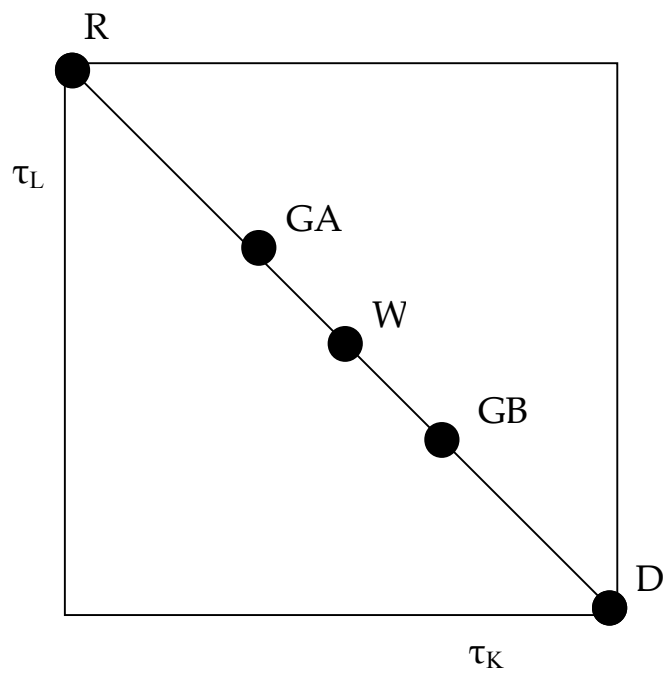


FIGURE 3

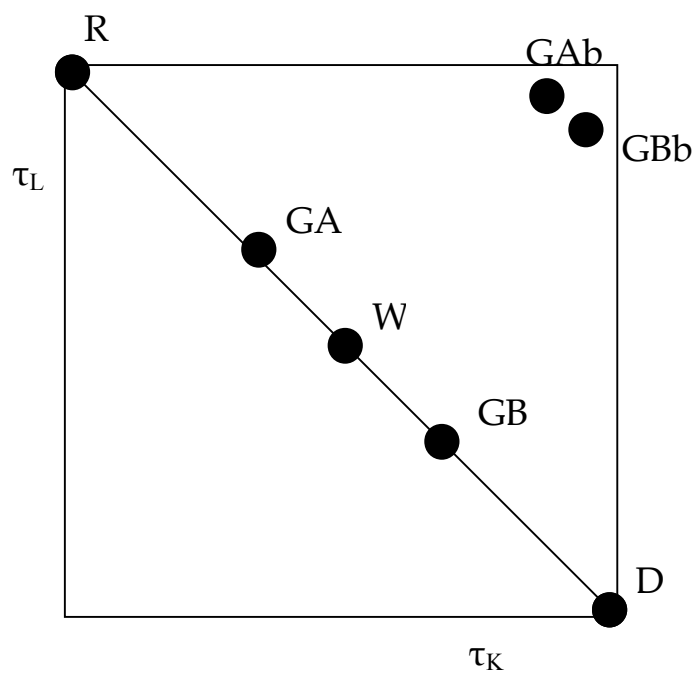


FIGURE 4

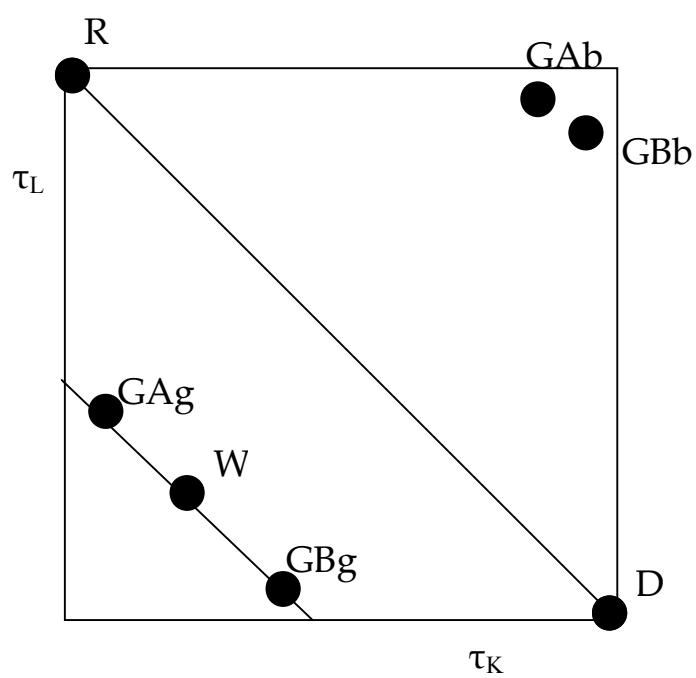


FIGURE 5

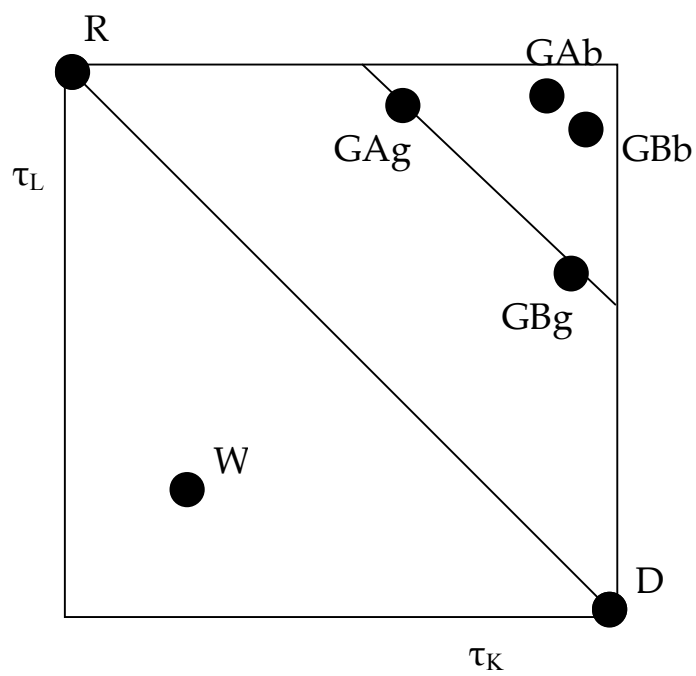


FIGURE 6